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Chaos in FPUT-like Systems

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Introduction

No unified picture of chaos in classical and quantum systems!

Classical Systems

- Defined with respect to sensitivity to initial conditions.
- Probed via Lyapunov exponents.

Quantum Systems

- Defined with respect to energy level statistics.
- Probed via OTOCs, operator growth, ...

Both can be probed via sensitivity to adiabatic deformations[1].

Our work: Develop an observable diffusion picture of classical chaos.

[1] C. Lim, K. Matirko, H. Kim, A. Polkovnikov, & M. Flynn. (2024). Defining classical and quantum chaos through adiabatic transformations.

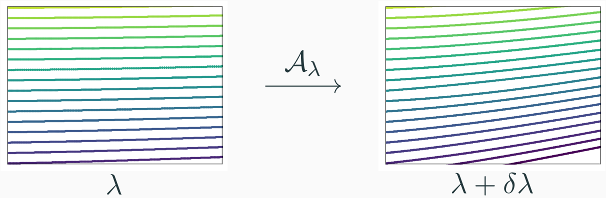
Adiabatic Gauge Potential[1] (AGP) in Classical Systems

- Consider an adiabatic change:

$$H(\lambda) \to H(\lambda + \delta\lambda)$$

- Deformations of trajectories in the phase space are generated by the AGP, $\mathcal{A}_\lambda(x, p)$.

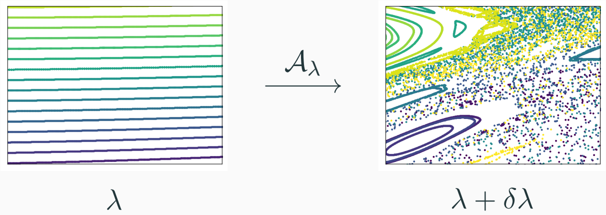
$$\frac{\partial x}{\partial \lambda} = \{x, \mathcal{A}_\lambda\},\ \frac{\partial p}{\partial \lambda} = \{p, \mathcal{A}_\lambda\}.$$



[1] M. Kolodrubetz, D. Sels, P. Mehta, & A. Polkovnikov. Geometry and non-adiabatic response in quantum and classical systems. Physics Reports, 697:1–87, 2017. ISSN 0370-1573.

AGP in Classical Systems

- Near-integrable trajectory: Small deformations.
- Chaotic trajectory: Large deformations.



- Claim: Variance of the AGP on a trajectory reveals its chaotic nature.

$$\chi_\lambda(T) = \frac{1}{T}\int_0^T dt\, \mathcal{A}_\lambda^2(x(t), p(t)) - \left(\frac{1}{T}\int_0^T dt\, \mathcal{A}_\lambda(x(t), p(t))\right)^2$$

An Analogy to Diffusion

Growth of the AGP variance can be interpreted as a diffusion process!

AGP	Diffusive Ensemble
$\mathcal{A}_\lambda(x(t), p(t))$	$X(t)$
$\partial_\lambda H(x(t), p(t))$	$V(t)$
$\chi_\lambda(T) \leftrightarrow \langle \partial_\lambda H(t) \partial_\lambda H(0) \rangle_c$	$\langle X^2(T) \rangle \leftrightarrow \langle V(t) V(0) \rangle$

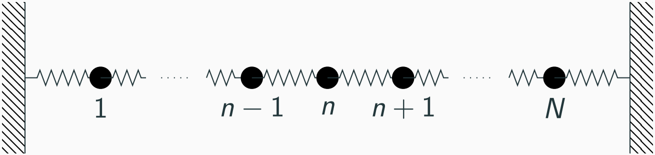
Fluctuation-dissipation relation: Growth of the AGP variance depends on the large time correlations of the perturbation.

AGP Diffusion

	$\langle \partial_\lambda H(t) \partial_\lambda H(0) \rangle_c$	$\chi_\lambda(T \to \infty)$
Integrable	Quasi-Periodic	Constant
Weak Chaos	$\sim 1/\lvert t \rvert^\gamma$	$\sim T^{2-\gamma}$
Strong Chaos	$\sim e^{-\lvert t/t_S \rvert}$	$\sim T$

- Integrable Systems: No diffusion.
- Weakly Chaotic Systems: Anomalous diffusion.
- Strongly Chaotic Systems: Normal diffusion.

β-Fermi-Pasta-Ulam-Tsingou (FPUT) System



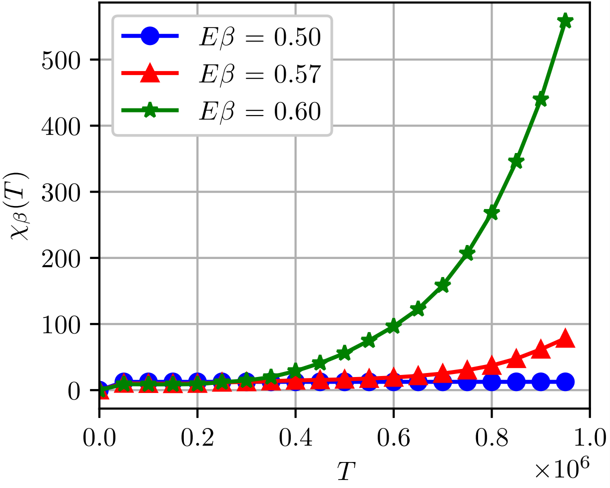
- Chain of oscillators with quartic interactions.

$$H = \sum_{n=0}^{N} \frac{p_n^2}{2} + \frac{1}{2}(q_{n+1} - q_n)^2 + \frac{1}{4}\beta(q_{n+1} - q_n)^4.$$

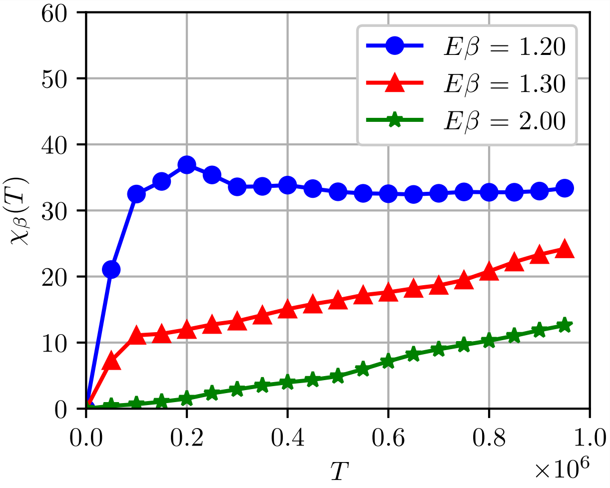
- Non-integrable system with long relaxation times.
- Long wavelength initial conditions.

β-FPUT System

Near-integrable to weak chaos transition

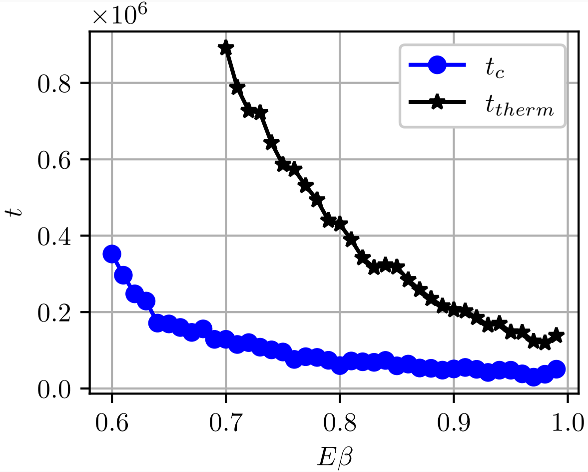


Weak to strong chaos transition



β-FPUT System

Onset time of chaos



Chaos is observed much sooner than thermalization.

Conclusions

- Adiabatic deformations can be used to probe chaos in classical systems.
- Observable diffusion can be used to characterize the nature of chaos.
- How is it related to Lyapunov exponents?
- Extension to other dynamical systems?

Thank you!

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